Effective Threshold Control in a Coded Communication System with Information Feedback Driven by Gaussian and Impulsive Noises

Vladimir Kazakov, Francisco Tejeda

Department of Telecommunications Engineering. SEPI-ESIME-IPN
Av. Instituto Politécnico Nacional s/n. 07738 México D.F., México
Phone (+5255) 57296000 ext. 54757
ftejeda@ipn.mx, vkazakov41@hotmail.com

Abstract. The expressions for the error probability and the average consumption of energy for the coded communication system with information feedback are obtained. A simple effective method of the threshold control at the feedback channel receiver is suggested when the Gaussian and Impulsive noises are present in the system. This method makes possible to avoid the useless consumption of energy in the coded system when the signal to noise ratio on the feedback channel is small and to improve the efficiency in other cases, taking into account the sum of both noises yields a quite destructive random process.

Keywords: Gaussian white noise, impulsive noise, coded communication system with information feedback.

1 Introduction

The Communication Systems with Information Feedback (CSIF) are within the communication systems with feedback channel [1-10]. The efficiency of the coded CSIF is characterized by the average consumption of energy and the error probability. We obtain the general expressions for these main characteristics of the CSIF. Furthermore, when the signal to noise ratio in the feedback channel is small, there is a useless consumption of energy, in other cases there is a chance to improve the characteristics of the CSIF. We suggest a simple and effective threshold control in the feedback channel receiver.

Let us describe shortly the logic operation of the coded CSIF [11]. At the transmitter part the message $a_k(\overline{1,N})$ is stores in memory, it is coded by the codeword A_k, and after that the word A_k is sent through direct channel. In the input of the direct channel receiver there is the codeword $\tilde{A} = A_k + S_d$ (here S_d is a disturbed combination occurred owing to the presence of noise in the direct channel). The space of the signals in the output of the direct channel decoder is $G = \{G_1, G_2, \ldots, G_N\}$, where $G_i(i = \overline{1,N})$ is the subspace of the codeword A_i and correspondingly of the message a_i. The received message a_i is stored in memory and is temporally not delivered to the recipient. On the basis of a_i the feedback channel coder generates the codeword B_i (generally from another code) and sends it through the feedback channel.

© L. Sánchez, O. Pogrebnyak and E. Rubio (Eds.)
Industrial Informatics
Research in Computing Science 31, 2007, pp. 33-42

In the input of the feedback channel decoder there is a disturbed codeword $\tilde{B} = B_k + S_f$ (here S_f is a disturbed combination occurred owing to the presence of noise in the feedback channel). The space of the signals in the output of the feedback channel decoder is $D = \{D_1, D_2, \dots, D_N\}$ where $D_j(j = \overline{1, N})$ is the subspace of the codeword B_j and correspondingly of the message a_j. In the transmitter side there is a comparison scheme. At the first input of this comparison scheme the message a_k is delivered from the memory block, at the second input the result of the decision of the feedback channel decoder is passed. When $\tilde{B} \in D_k$ occurs, there is a coincidence between the message sent through the direct channel and the message received in the feedback channel receiver. In this case, the comparison scheme produces an acknowledgement control signal in order to send the next message through the direct channel. The acknowledgement control signal is received by the direct channel receiver which sets free the message stored in its memory and it is delivered to the recipient. When $\tilde{B} \notin D_k$ there is no coincidence between the index of the sent and received messages in the input of the comparison scheme. In this case, a rejection control signal is generated by the comparison scheme. The rejection control signal and the same message a_k are sending from the transmitter part. The reception of the rejection control signal blocks out the delivery of the received message from the memory of the direct channel receiver to the recipient. This process goes on until the indexes in both inputs of the comparison scheme are the same and the system passes to the transmission of the next message.

2 New random process

Let us consider *RC* filters are in the receivers of both direct and feedback channels. Let us assume that in the input of the *RC* filter the following process is present on both channels

$$\xi_i(t) = \theta s_{in_i}(t) + n_i(t) + m_i(t), \tag{1}$$

where θ is a parameter of presence of the signal (i.e., θ=0 the signal is no present; θ=1 the signal is present), the index i indicates the channel (it could be direct or feedback), $s_{in}(t)$ is the input rectangular signal with amplitude A_i, $n_i(t)$ is the Gaussian process and $m_i(t)$ is the impulsive process. The resulting output process is expressed

$$x_i(t) = s_{out_i}(t) + x_{1_i}(t) + x_{2_i}(t), \tag{2}$$

where

$$s_{out_i}(t) = A_i(1 - e^{-\alpha_i t}), \tag{3}$$

$$\alpha_i = \frac{1}{R_i C_i}, \tag{4}$$

α_i is the constant of the *RC* filter, $s_{out}(t)$ is the output signal, $x_1(t)$ is a Gaussian random variable and $x_2(t)$ is a random variable described by the Beta distribution [12]. The Probability Density Function (PDF) of the Gaussian random variable is well known

$$w_1(x_1) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(x_1-\theta s_{out}(T))^2}{2\sigma_1^2}}, \tag{5}$$

where $T=T_d=T_f$ is the duration of the signals in both channels, σ_1^2 is the variance in stationary state and is known; its mathematical expectation depends on the parameter θ, it means, if $\theta=0$, $s_{out}(T) = 0$; if $\theta=1$, $s_{out}(T) = A_i(1 - e^{-\alpha_i T})$. The Beta distribution is defined by

$$w_2(x_2) = \frac{x_2^{\lambda_1-1}(1-x_2)^{\lambda_2-1}}{B(\lambda_1,\lambda_2)}, 0 \le x_2 \le 1, \tag{6}$$

where λ_1, λ_2 are the Poisson's flow intensities, $B(\cdot)$ is the Beta function. The mathematical expectation is

$$\langle x_2 \rangle = \frac{\lambda_1}{\lambda_1 + \lambda_2}, \tag{7}$$

and its variance

$$\sigma_2^2 = \frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_2)^2(\lambda_1+\lambda_2+1)}. \tag{8}$$

In order to get the PDF of the sum of the Gaussian and impulsive noises the convolution theorem is applied

$$w(x) = \int_{-\infty}^{\infty} w_1(x_1 - x_2)\, w(x_2) dx_2 . \tag{9}$$

The resulting PDF depends on many parameters inherit from the two random processes. However, the Poisson's flow intensities have a strong influence in the shape of the new PDF, as it could be symmetrical ($\lambda_1 = \lambda_2$) or asymmetrical ($\lambda_1 \neq \lambda_2$). The mathematical expectation of this new PDF is determined by

$$\langle x \rangle = \theta s_{out}(T) + \langle x_2 \rangle, \tag{10}$$

and the variance is

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 . \tag{11}$$

3 General expressions

We analyze the coded CSIF with the following assumptions: 1) the number of possible repetitions of the message is equal to infinity; 2) the transmission of the acknowledgement and rejection control signals is ideal; 3) the coding schemes in both channels are not changed in the repetition process.

We note the principal difference between the problems solved by both decoders: the direct channel decoder must solve the problem of determining one among many possible words; therefore, in its output any word $A_l(\overline{1,N})$ can appear; the feedback channel decoder must solve the problem of the detection of one known word B_k because the index k is a priori known in the transmitter part of the system. In other words, the feedback channel decoder solves the following binary problem: $\tilde{B} \in D_k$ or $\tilde{B} \notin D_k$ (Fig. 1). Taking into account this difference, one can characterize any cycle of the message a_k transmitted by the following three probabilities:

1) the probability of correct reception $\hat{p}$

$$\hat{p} = P_k[\tilde{A} \in G_k]P_k[\tilde{B} \in D_k]\,, \tag{12}$$

where $P_k[\cdot]$ means the probability of the event $[\cdot]$ if the messages a_k and b_k were transmitted in the direct and feedback channels correspondingly.

2) The error probability $\hat{q}$

$$\hat{q} = \sum_{i \neq k} P_k[\tilde{A} \in G_i]P_i[\tilde{B} \in D_k]\,, \tag{13}$$

3) The probability of error detection $\hat{u}$

$$\hat{u} = P_k[\tilde{A} \in G_k]\sum_{i \neq k} P_k[\tilde{B} \in D_i] + \sum_{i \neq k} P_k[\tilde{A} \in G_i]P_i[\tilde{B} \in D_i]\,. \tag{14}$$

It is clear that

$$\hat{p} + \hat{q} + \hat{u} = 1\,. \tag{15}$$

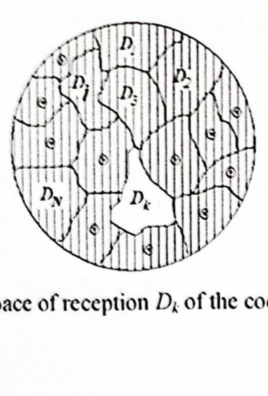

Fig. 1. The space D and the subspace of reception D_k of the codeword B_k.

However, we must specify these general expressions. We extend the coding formulas from a simple direct channel to the coded CSIF [13]. Therefore, the probability of correct reception $\hat{p}$ for any lineal and systematic code is

$$\hat{p} = \sum_{m=0}^{t_d} \binom{m}{n_d} q_d^m (1 - q_d)^{n_d - m} \sum_{m=0}^{t_f} \binom{m}{n_f} q_{f_1}^m (1 - q_{f_0})^{n_f - m}, \tag{16}$$

$$q_d = 0.5 \int_h^{+\infty} w_d(x) dx + 0.5 \int_{-\infty}^{h} w_d(x - \langle x \rangle) dx, \tag{17}$$

$$q_{f_0} = \int_{\beta_0}^{+\infty} w_f(x)\, dx, \tag{18}$$

$$q_{f_1} = \int_{-\infty}^{\beta_0} w_f(x - \langle x \rangle)\, dx, \tag{19}$$

where q_d is the probability of the symbol error in the direct channel; q_{f_0} and q_{f_1} are the error probabilities for the zero and one symbols in the feedback channel; where h is the threshold decisions in the direct receiver; β_0 is the "initial" threshold decision in the feedback channel receiver; $w_d(\cdot)$ and $w_f(\cdot)$ are the PDF's in the output of the RC filters in the direct and feedback receivers; t_d and t_f are the maximum capacity that the code schemes have to correct errors in the direct and feedback channels; m is the amount of errors that the codeword has; n_d and n_f are the length of the codeword in the direct and feedback channels. The expression for the error probability per codeword is

$$\hat{q} \le (N_d - 1) \sum_{m=0}^{t_d} \binom{m}{n_d} q_d^{n_d - m} (1 - q_d)^{n_d - d_d + m} \sum_{m=0}^{t_f} \binom{m}{n_f} q_{f_1}^{n_f - m} (1 - q_{f_0})^{n_f - d_f + m}, \tag{20}$$

where d_d and d_f are the minimum code distances in the direct and feedback channels; $N_d = 2^{k_d}$ is the number of codewords and k_d is the number of informative symbols in the codeword in the direct channel. The error probability per codeword of the CSIF with an infinity number of repetitions is determined by the formula:

$$Q = \hat{q} + \hat{u}\hat{q} + \hat{u}^2\hat{q} + \cdots = \frac{\hat{q}}{1 - \hat{u}} = \frac{\hat{q}}{\hat{p} + \hat{q}}. \tag{21}$$

In the same manner, one can obtain the formula for the average consumption of energy $\langle E \rangle$ of one codeword transmitted

$$\langle E \rangle = \frac{n_d E_d + n_f E_f}{\hat{p} + \hat{q}}, \tag{22}$$

where E_d and E_f are the energy of any elementary signal in the direct and feedback channels. We divide (22) by the white noise spectral density $N_0/2$, thus

$$\langle Z^2\rangle = \frac{n_d z_d^2 + n_f z_f^2}{\hat{p} + \hat{q}}, \tag{23}$$

where $z_d^2 = 2E_d/N_0$ and $z_f^2 = 2E_f/N_0$ are the square of the signal to noise ratio (SNR) in both direct and feedback channels.

The expressions (21) and (23) describe the principal characteristics of the coded CSIF regards to a codeword. But as it is known, different codes have a different efficiency with the same SNR. It means that in order to make a correct comparison among the efficiency of different codes in the coded CSIF, it is necessary to recalculate the expressions (21) and (23) per one symbol. Using the methodology described in [14], we can define the error probability equivalent per one informative symbol

$$q = 1 - (1 - Q)^{\frac{1}{k_d}}. \tag{24}$$

In order to calculate the average consumption of energy per one informative symbol, one can write down the following formula

$$\langle z^2\rangle = \frac{\langle Z^2\rangle}{k_d} = \frac{n_d z_d^2 + n_f z_f^2}{k_d(\hat{p} + \hat{q})}. \tag{25}$$

The formulas (24) and (25) are used for all calculations, since they provide a possibility to characterize the efficiency of any CSIF by the function $q(<z>)$. It is very useful to introduce another function

$$q_0(z_0) = q_d(z_d), \tag{26}$$

where q_0 is the error probability in a communication system with one channel which uses amplitude modulation and the a priori probabilities are equal.

4 Example

For our goal it is enough to choose a simple variant of the coded CSIF. Namely, we suppose that the code (7, 4, 1) is used in both channels (here n_d=n_f=7, k_d=4, t_d=t_f=1). Besides this, we must take into account others parameters: the *RC* factor α = 1, the duration of the signal T = 1, the impulsive noise amplitude is 1. We choose the Poisson's flow intensities λ_1 = λ_2 = 1 since the simplest case for illustration is a symmetrical one. Nonetheless, these parameters could take others values. The results of calculations following the formulas (24) and (25) are presented in Fig. 2. There are two curves: 1) the curve *a* represents the function $q_0(z_0)$; 2) the curve *b* consists of two parts b_1 and b_2; it characterizes the efficiency of the coded CSIF $q(<z>)$ when z_d = 2 and the value of z_f is widely changed ($0 \leq z_f \leq 3.5$), these values are marked by circles.

The first part b_1 is related to the case $z_f \leq 1.1$ (or $z_f < z_d$). As one can see, when the parameter z_f increases from 0 to 1.1, the error probability q decreases a little ($-0.98 \leq \log q \leq -0.66$). Furthermore, we note that the average consumption of energy $<z>$ is quite big in comparison with the values z_d and z_f. The reason of this effect is connected with a fairly error probabilities q_{t_0} and q_{f_1} which leads to the fairly probability of repetitions $\hat{u}$ as well as the increasing average number of repetitions. It is clear that this regime is very bad and undesirable owing to the huge consumption of energy and the enormous amount of errors in the auxiliary feedback channel. The efficiency of the coded CSIF in this regime under consideration is so much worse than the efficiency of the system with one channel which uses amplitude modulation and has the same parameter values because the location of the curve b_1 is higher than the location of the curve a.

The second part b_2 is related with the case $z_f > 1.1$. Here the situation is changed mainly: when z_f increases then the error probability q decreases quickly. The coded CSIF efficiency in this regime is higher than the system efficiency with one direct channel due to the fact that the location of the curve b_2 is lower than the location of the curve a. It is important to note that the calculations of the coded CSIF efficiency with others codes leads to the same shape of the function $q(<z>)$.

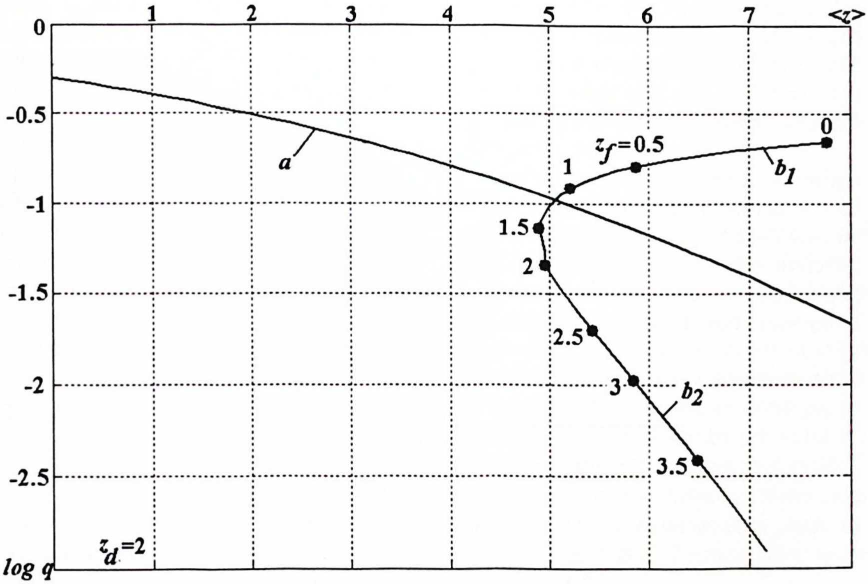

Fig. 2. The coded CSIF efficiency (curves b_1 and b_2) with the elementary signals reception in the feedback channel with initial threshold.

5 The proposed method

In order to avoid the regime describe by the curve b_l, it is theoretically necessary to increase the subspace D_k (see Fig. 1). In this case, the probability of the correct reception of the codeword B_k (correspondingly of the message a_k) must be increased also. This effect leads to decrease the number of repetitions and the consumption of energy. Hence, we need to change the feedback channel threshold β in a wide range so that we could obtain an optimal regime of system functioning. Now let us rewrite the formulas for the probabilities $\hat{p}$ and $\hat{q}$ taking into account an arbitrary threshold β in the feedback channel receiver. Then instead of (18) and (19), we have

$$q_{f_0} = \int_{\beta_0-\beta}^{+\infty} w_f(x)dx\,, \tag{27}$$

$$q_{f_1} = \int_{-\infty}^{\beta_0+\beta} w_f(x-\langle x\rangle)dx\,. \tag{28}$$

Putting (27) and (28) into (16) and (20), let us apply these formulas for the efficiency calculation of the CSIF with codes (7, 4, 1) on both channels. Like in Fig. 2, let us choose the same parameter values for the system, $z_d = 2$ and $z_f = 0.5, 1.0, 1.5, 2.0, 2.5, 3$. Changing the threshold β ($-\infty \le \beta \le +\infty$) and applying the recalculation procedure (24) and (25) per one informative symbol, we obtain the family of curves $q(<z>)$ depicted in Fig. 3.

The curve c is the envelope of this family and characterizes the optimal regime of functioning for the chosen parameter $z_d = 2$. The crossing points of the family curves $q(<z>)$ with curve b are marked by circles. As one can see, the initial thresholds are optimal near the value $z_f = 2.5$. In other cases, it is necessary to choose different values for the thresholds. If $z_f < 1.1$, the receiver must be slightly open for an expected codeword. The concrete values of the threshold can be determined as points of contact of the family curves with the envelope curve c as in table 1 is illustrated.

Table 1. The concrete values of the threshold regarding z_f in the envelope curve c.

z_f	$\mp\beta$
0.5	0.6
1.0	0.4
1.5	0.2
2	0.1
2.5	0

In table 1, the double signs mean the negative value for the symbol 1 and the positive value for the symbol 0. Such choice of thresholds avoids the issue discussed in Fig. 2: the bad regime with huge consumption of energy (compare the envelope c higher the point $z_f = 1.1$ and the curve b_l). The family curves $q(<z>)$ with $z_f \ge 1.1$ demonstrate that it is necessary to decrease the subspace D_k. It means that the signs of the thresholds must be invert in the points of contact of the family curves with the envelope c. In this case we have: $z_f = 3$, $\pm\beta = 0.1$

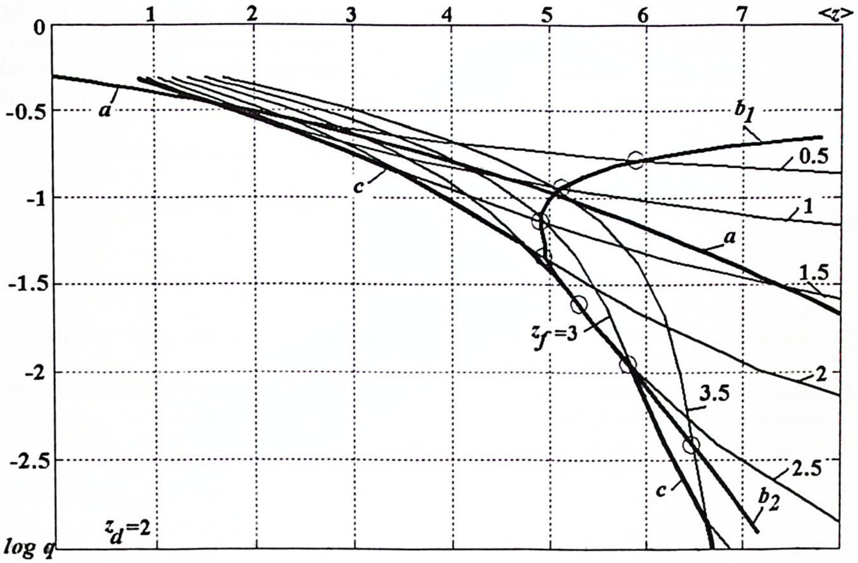

Fig. 3. The coded CSIF with several values of the parameter z_f when the thresholds in the feedback channel receiver are varied according to the proposed method.

The shape of the envelope c demonstrates: 1) the application of the proposed method provides a possibility of improving the CSIF efficiency in comparison with the variant of initial threshold. Actually, if $z_f < z_d$ (to be exact here $z_f < 1.1$) the undesirable regime with huge consumption of energy is avoided; if $z_f \geq z_d$, the error probability q decreases with the same consumption of energy. 2) if $z_f < z_d$ (to be exact here $z_f < 1.1$) the CSIF efficiency is worse than the efficiency of a communication system with one direct channel since the curve c is higher than curve a or curve $q_0(z_0)$.

It is necessary to emphasize that all features above mentioned are valid for any type of systematic code used in the CSIF. Nevertheless, the optimal thresholds β values and the corresponding values of q and <z> must be others. Although there exist others parameters which determine the performance of the system also, such as: Poisson's flow intensities, *RC* factor α, duration of the signal T, impulsive noise amplitude, etc.

6 Conclusion

A simple practical method of the threshold control in a feedback channel receiver is suggested when the coded CSIF is driven by Gaussian and impulsive noises. Despite of the fact that sum of Gaussian and impulsive noises yields in the output of the *RC* filter in both channel receivers a quite destructive random process, an improvement of the efficiency of the system was achieved when the threshold decision in the feedback channel receiver was varied in a wide range. In spite of the optimal performance of the coded CSIF depends on many parameters, it was found.

References

1. Chang, S.S.L.: Improvement of two-way communication by means of feedback. IRE Intern. Conv. Record, (1961) Pt. 4
2. Kazakov, V.A.: Noise immunity of a communication system with comparison. Telecommunications, Vol. 22, No. 2, (1968) 18-23
3. Kazakov, V.A.: Noise immunity of a memoryless comparison system with a limited number of repetitions. Telecommunications, Vol. 22, No. 8, (1968) 13-19
4. Kanevskij, Z.M.: Transmission of message by information feedback. Sov. Radio, Moscow (1968) – in Russian
5. Klove, T., Korzhik, V.: Error detected code. General theory and their application in feedback communication systems. Kluver academic publisher, Boston, (1995)
6. Ooi, J.M.: A framework for low-complexity communication over channels with feedback. Ph.D. dissertation, Massachusetts Institute of Technology, (1998)
7. Ooi, J.M.: Coding for channels with feedback. Kluver academic publisher, Massachusetts (1998)
8. Wicker, S.: Error control systems for digital communication and storage. Prentice Hall, New Jersey (2002)
9. Tatikonda, S. C.: Control under communication constraints. Ph.D. dissertation, Massachusetts Institute of Technology, (2000)
10. Klein, T. E.: Capacity of Gaussian noise channels with side information and feedback. Ph.D. dissertation, Massachusetts Institute of Technology, (2001)
11. Bargh, M. S.: Coding strategies for channels with feedback. Ph.D. dissertation, Eindhoven University of Technology, Netherlands, (1999)
12. Stuart, A., Ord, K.: Kendall's advanced theory of statistics. Vol. 1, Arnold, sixth edition, London, (1994) p. 200
13. Lin, S., Costello, D.: Error control coding. Prentice Hall, New Jersey (1983)
14. Blokh, E.L.: Noise immunity of a decision feedback communication system. Science academy URSS, (1963) – in Russian